Aircraft Performance in Two Charts
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AirVenture 2018
Oshkosh, Wisconsin
July 23 – 29, 2018
First Some Background
Thrust Horsepower Required To Maintain Level Flight

\[ P_r = \frac{\sigma \rho_{ssl}}{2} f V^3 + \frac{2}{\sigma \rho_{ssl} \pi \epsilon} \left( \frac{nW}{b} \right)^2 \frac{1}{V} \]

parasite effective induced

\[ EAS = \sqrt{\sigma} V \quad \text{or} \quad V = \frac{EAS}{\sqrt{\sigma}} \]

\[ P_r = \frac{\rho_{ssl}}{2} f \frac{EAS^3}{\sqrt{\sigma}} + \frac{2}{\rho_{ssl} \pi \epsilon} \left( \frac{nW}{b} \right)^2 \frac{1}{\sqrt{\sigma} EAS} \]

parasite effective induced

\[ P_r = \text{Constant} f \frac{EAS^3}{\sqrt{\sigma}} + \text{Konstant} \frac{1}{\sqrt{\sigma} EAS} \]

parasite effective induced
\[ P_r = \text{Constant} \cdot f \frac{EAS^3}{\sqrt{\sigma}} + \text{Konstant} \cdot \frac{1}{\sqrt{\sigma} EAS} \]

\[ (\text{EAS}_{L/D_{\text{max}}})^2 = \frac{2}{\rho_{\text{SSL}}} \frac{W}{b} \frac{1}{\sqrt{\pi fe}} \]

\[ = \text{Constant} \cdot \frac{1}{\sqrt{f}} \]
\[ P_r = \text{Constant} \cdot \frac{EAS^3}{\sqrt{\sigma}} + \text{Konstant} \cdot \frac{1}{\sqrt{\sigma} \cdot EAS} \]

\[ (\frac{EAS_{L/D_{\text{max}}}}{D_{\text{max}}})^2 = \frac{2}{\rho_{ssl}} \cdot \frac{W}{b} \cdot \frac{1}{\sqrt{\pi} \cdot f_e} \]

\[ = \text{Constant} \cdot \frac{1}{\sqrt{f}} \]

\[ EAS_{Pr_{\text{min}}} = 0.76 \cdot EAS_{L/D_{\text{max}}} \]
Thrust Horsepower

EAS

Induced

Parasite

Total

Backside

Frontside

EAS_{Pr_{min}}

EAS_{L/D_{max}}
Aircraft Modelled

Normally Aspirated
Single Engine Light Retractable General Aviation Aircraft
285 BHP IO-520 Continental Engine
Three Blade Propeller 80" Diameter
Gross Weight 3300 lbs
Equivalent Parasite Drag From Flight Test
Propeller Efficiency Corrected For Spinner Area
BHP Corrected For Sea Level Manifold Pressure Loss Based On Flight Test

Configurations Considered

Clean
Gear Down Flaps Zero
Gear Down Flaps 20 degs
Gear Down Flaps 32 degs

Altitudes Considered
Sea Level
5000 ft
10,000 ft
Sea Level Performance
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

Thrust Horsepower (Thp)

EAS (mph)

Sea Level IO-520BB

Increasing Alpha

Decreasing Alpha

55%

60%

65%

70%

75%

80%

81%

Power On

Stall

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Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

Sea Level IO-520BB

Power On Stall

Increasing Alpha

Decreasing Alpha
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

Sea Level
IO-520BB

Increasing Alpha

Decreasing Alpha

V\_max
V\_\gamma\_max
V\_R/C\_max
V\_L/D\_max

P\_available
P\_r
C\_lean = 3.125
P\_r\_Gear + Flaps 0 \ f = 7.593
Pr Gear + Flaps 20 \ f = 9.829
Prop Efficiency
V(R/C\_max) \ V\_y
V(Gamma Max) \ V\_x
V(L/D\_max)

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Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

Sea Level
IO-520BB

Increasing Alpha
Power On
Stall Line

Decreasing Alpha

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Standard Sea Level on a Standard Day

What are the take aways?

- The intersections of the thrust horsepower available and the thrust horsepower required curves represent possible level flight states (speeds).
  
  Here only a single intersection occurs – the high speed intersection. 
  
  Stall occurs before the low speed ‘intersection’.

- The vertical distance between the thrust horsepower available and the thrust horsepower required curves represents the available rate of climb.

- Propeller efficiency decreases with decreasing airspeed.

  Thus, the propeller efficiency for the V-speeds, EAS\(_L/D_{\text{max}}\), EAS\(_R/C_{\text{max}}\) and EAS\(_{\text{vmax}}\) decrease with decreasing airspeed.

With Increasing Parasite Drag:

- The equivalent airspeed ‘band’ for positive rate of climb decreases;

- The equivalent airspeed for maximum lift to drag ratio, i.e., the speed for best glide, EAS\(_L/D_{\text{max}}\) decreases;

- The equivalent airspeed for maximum rate of climb speed, EAS\(_R/C_{\text{max}}\) decreases;

- The available positive rate of climb decreases;

- The equivalent airspeed for maximum climb angle speed, EAS\(_{\text{vmax}}\) decreases.
What Are The Effects of Altitude?
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

- Power Available \( P_{\text{available}} \) = 3.125
- Power Cleared \( P_{\text{clean}} \) + Flaps 0 \( P_{\text{clean}} + Flaps 0 \) = 7.593
- Power On \( P_{\text{op}} \) + Eff
- Stall

\[ V_{\text{max}}(R/C_{\text{max}}) \]
\[ V_{\text{y}}(G_{\text{am}}a_{\text{max}}) \]
\[ V_{\text{x}}(L/D)_{\text{max}} \]
Power Available - Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

- Prop Efficiency: \( V(R/C_{max}) \), \( V_y \), \( V(Gamma \: Max) \), \( Vx \)

- Increasing Alpha: 50%, 60%, 65%, 70%, 75%, 80%, 81%
- Decreasing Alpha: 50%, 60%, 65%, 70%, 75%, 80%, 81%

Power On - Stall Line: \( V_{R/C_{max}} \), \( V_{max} \), \( V_{L/D_{max}} \)

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In terms of the equivalent airspeed, EAS:

- The EAS_{L/D_{max}} does not change with altitude for any configuration;
- The power required curves move up and to the left;
- The power available curve moves down and to the left;
- The result is a decrease in the:
  - available equivalent speed band for positive rate of climb;
  - the available rate of climb.
- Here, at full power, two intersections occur with the gear and flaps full – the high speed intersection and a low speed intersection. The low speed ‘intersection’ occurs before stall.

With Increasing Parasite Drag:

- The equivalent airspeed for maximum lift to drag ratio, EAS_{L/D_{max}} decreases;
- The equivalent airspeed for maximum rate of climb, EAS_{R/C_{max}} decreases;
- The equivalent airspeed for maximum climb angle, EAS_{θ_{max}} decreases;

Compared to Sea Level at 5000 ft:

- The equivalent airspeed band for positive rate of climb decreases further;
- The maximum rate of climb speed, EAS_{R/C_{max}} decreases for all configurations;
- The speed for maximum climb angle, EAS_{θ_{max}} increases for all configurations;
- Propeller efficiency increases at any given EAS;
  - Thus, the propeller efficiency for the V-speeds EAS_{L/D_{max}} and EAS_{R/C_{max}} decreases with decreasing airspeed;
  - However, the propeller efficiency for the V-speed EAS_{θ_{max}} increases slightly.

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Altitude Effects 10,000
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP

Thrust Horsepower (Thp)

EAS (mph)

Power On
Stall

Increasing Alpha
Decreasing Alpha

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Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP

Power On-Stall

EAS (mph)

Thrust Horsepower (Thp)

5000 ft

IO-520BB

Decreasing Alpha

Increasing Alpha

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Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 208 BHP

Power On-Stall

EAS (mph)

Thrust Horsepower (Thp)

10,000 ft

IO-520BB

Decreasing Alpha

Increasing Alpha

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Power Available - Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP

Power On-Stall

EAS (mph)

Thrust Horsepower (Thp)

5000 ft

IO-520BB

Decreasing Alpha

Increasing Alpha

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Power Available - Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 208 BHP

Power On-Stall

EAS (mph)

Thrust Horsepower (Thp)

10,000 ft

IO-520BB

Decreasing Alpha

Increasing Alpha

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Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP

Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 208 BHP
Again, the available EAS `band' for positive rate of climb further decreases with the increase in altitude. It also decreases as the aircraft is `dirtyed' up. Available positive rate of climb decreases further with altitude decrease. It also decreases as parasite drag is added.

At 10,000 ft the maximum rate of climb speed, EASR/Cmax further decreases for all configurations compared to lower altitudes. It also decreases as parasite drag is added.

The speed for maximum climb angle, EASgmax, increases at 10,000 ft compared to lower altitudes. But, it decreases as parasite drag is added.

Propeller efficiency increases at any given EAS compared to that at sea level. It also decreases with decreasing airspeed.

Thus, the propeller efficiency for the V-speeds for EASL/Dmax and EASR/Cmax decreases with decreasing airspeed. However, the propeller efficiency for EASgmax increases slightly.

What Are The Take Aways?

- In terms of the equivalent airspeed, EAS, the power required curves move further up and to the left while the power available curve moves down and to the left. The result is a decrease in the available speed range for positive rate of climb and the actual available rate of climb.
- Here, at full power, two intersections occur -- a high and a low speed intersection. Stall occurs below the lower speed `intersection' with gear and full or approach flaps extended.
- In this case the aircraft cannot maintain steady level flight before it stalls. In the clean configuration stall occurs above the lower speed. Hence, the aircraft stalls first.
- This is like the turbine `coffin' corner.

The EASL/Dmax does not change with altitude for any configuration.

Again, the vertical distance between the thrust horsepower available and the thrust horsepower required curves represents the available rate of climb.

Available positive rate of climb decreases further with altitude decrease. It also decreases as parasite drag is added.

At 10,000 ft the maximum rate of climb speed, EASR/Cmax further decreases for all configurations compared to lower altitudes. It also decreases as parasite drag is added.

The speed for maximum climb angle, EASgmax, increases at 10,000 ft compared to lower altitudes. But, it decreases as parasite drag is added.

Propeller efficiency increases at any given EAS compared to that at sea level. It also decreases with decreasing airspeed.
Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 278 BHP

- P_{available} = 3.125
- P_{Gear} + Flaps 0 \ f = 7.593
- P_{Gear} + Flaps 20 \ f = 9.829
- P_{Gear} + Flaps 32 \ f = 12.712

- Prop Efficiency
- V(R/C_{max}) \ Vy
- V(Gamma Max) \ Vx
- V(L/D)_{max}

Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 242 BHP

Power Available-Power Required, Bank Angle 0 deg, Weight 3300 lbs, 100% Power Available 208 BHP
Turning Flight
Thrust Horsepower

Velocity

Induced

Parasite

Total

Parasite

Induced
Thrust Horsepower Required To Maintain Level Turning Flight

\[ P_r = \frac{\rho_{SSL}}{2} \frac{f EAS^3}{\sqrt{\sigma}} + \frac{2}{\rho_{SSL}} \frac{1}{\pi e} \left( \frac{nW}{b} \right)^2 \frac{1}{\sqrt{\sigma EAS}} + \frac{1}{\sqrt{\sigma EAS}} \cos^2 \phi \]

In a level turn the load factor \( n = 1/ \cos \phi \) where \( \phi \) is the bank angle.

Hence,

\[ P_r = \frac{\rho_{SSL}}{2} \frac{f EAS^3}{\sqrt{\sigma}} + \frac{2}{\rho_{SSL}} \frac{1}{\pi e} \left( \frac{W}{b} \right)^2 \frac{1}{\sqrt{\sigma EAS}} \cos^2 \phi \]

which using the same procedures as previously reduces to

\[ P_r = \text{Constant} \frac{f EAS^3}{\sqrt{\sigma}} + \text{Konstant} \frac{1}{\sqrt{\sigma EAS}} \frac{1}{\cos^2 \phi} \]

Similarly, the speed for maximum lift to drag ratio increases

\[ (EAS_{L/D_{\text{max}}})^2 = \frac{2}{\rho_{SSL}} \frac{nW}{b} \frac{1}{\sqrt{\pi fe}} \]

\[ = \text{Constant} \frac{1}{\sqrt{f}} \frac{1}{\cos \phi} \]

\[ EAS_{L/D_{\text{max}}} = \sqrt{\text{Constant} \frac{1}{\sqrt{f}} \frac{1}{\cos \phi}} \]
Operational Considerations
More Information

www.nar-associates.com/technical-flying/technical-flying.html
Power Available Power Required Bank Angle 0 deg
Weight 3000 lbs 100% Power Available 255 BHP

- Prop Efficiency
- V(R/C_\text{max}) \text{ Vy}
- V(\text{Gamma Max}) \text{ Vx}
- V(L/D)_{\text{max}}
Power Available Power Required Bank Angle 30 deg

Weight 3300 lbs 100% Power Available 208 BHP

10,000 ft 30°
IO-520BB

Power Available
Pr Clean
Pr Gear + Flaps 0
Pr Gear + Flaps 20
Pr Gear + Flaps 32
Prop Efficiency

EAS (mph)
Thrust Horsepower (Thp)