Propeller Efficiency

Takeoff

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Introduction

From a previous article (Propeller Efficiency - Rule of Thumb) recall that propeller efficiency was characterized by two nondimensional parameters, the advance ratio, $J$, and the power coefficient, $C_p$, given by

$$J = \frac{V}{ND} \quad \text{and} \quad C_p = \frac{\text{BHP}}{\rho N^3 D^5}$$

where $V$ is the true airspeed, $N$ is the propeller RPM, $D$ is the diameter of the propeller, $\rho$ (rho) is the local density and BHP is, of course the brake horsepower.

From that discussion we developed A Simplified Rule of Thumb (ASROT)† that allowed estimation of the appropriate RPM for a given cruise true airspeed (KTAS) and showed how to use the cruise performance graph in the pilot operating handbook (POH) to ‘calibrate’ the result for a given propeller. For the McCauley C76 propeller the result was that the ratio of the RPM to KTAS should be about 14.5 or

$$\frac{\text{RPM}}{\text{KTAS}} = 14.5$$

Takeoff

In that previous article we also noted that if a constant speed propeller was optimized for efficiency during cruise, then the propeller efficiency during takeoff is significantly nonoptimal. Let’s take a look at this.

Consider a McCauley C76 three bladed propeller for an E33A equipped with an IO-520 engine at sea level on a standard day. Maximum RPM is constant at 2700 RPM, and propeller diameter is fixed. From the equations above we note that the advance ratio, $J$, increases directly with the takeoff velocity, $V$. Similarly, brake horsepower (BHP) is also

†Pronounced AS ROT
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constant at 285 BHP. The density, $\rho$, is also constant. As a result, the power coefficient, $C_p$, is constant at

$$C_p = \frac{\text{BHP}}{\rho N^3 D^5} = \frac{(550)(285)}{(0.002378)(2700/60)^3(80/12)^5} = 0.0549$$

where the 550, 60 and 12 are conversion factors and 0.002378 is the density at sea level in appropriate units. Similarly, the advance ratio, $J$, is

$$J = \frac{V}{ND} = \frac{1.69V}{(2700/60)(80/12)} = \frac{1.69}{300} V = 0.00563 V \quad V \text{ in KTAS}$$

The propeller efficiency from 10 to 110 KTAS is shown as the dashed red line on the bare propeller map in Figure 1. The black $\times$ indicates 70 KTAS, which is a typical rotation velocity for the E33A. The cruise results are also shown for comparison. From Figure 1 we see that the propeller efficiency during the takeoff run is significantly less than that during typical cruise conditions, represented by the colored dots. In fact, at 70 KTAS the propeller efficiency is approximately 0.65 (65%).

Altitude effect

An interesting question is: What is the effect of altitude on propeller efficiency during the takeoff run? It turns out that there is not much effect of altitude on the power coefficient. Specifically, the change in brake horsepower is approximately proportional to the change in density. Thus, the effect of altitude (density) on $C_p$ a quite small. Using the altitude performance curves for the IO-520BB† the BHP at 10,000ft on a standard day is 200 BHP

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Figure 1. McCauley C76 propeller map.


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at 2700 RPM. Thus, the power coefficient $C_p = 0.052$, which is very close to that at sea level. However, the advance ratio increases because the true airspeed at 10,000ft is 81.5 KTAS for an equivalent airspeed of 70 KTAS at sea level. These results are shown as the red long dashed line in Figure 1. The black + sign on the line corresponds to an equivalent airspeed of 70 KEAS (81.5 KTAS) at 10,000 ft.†

The result is a small increase in propeller efficiency at the rotation speed of 70 KEAS (knots equivalent airspeed) compared to that at sea level, i.e., from approximately 0.65 to 0.715. However, the takeoff ground run will still be longer because of the higher true airspeed at rotation at 10,000 ft.

**Turbonormalization**

Here, the advance ratio is the same as for the normally aspirated aircraft because the rotation velocity is the same. However, 10,000 ft is below the critical altitude for the turbonormalized engine. Hence, the full sea level power of 285 BHP is available. Because the density is lower, the power coefficient increases to $C_p = 0.0744$ compared to the sea level value of 0.0549. The green solid line in Figure 1 shows the result. Again, a black + sign identifies the rotation speed of 70 KEAS. Compared to the normally aspirated IO-520BB, the TNIO-520BB suffers a decrease in propeller efficiency, i.e., from approximately 0.715 to 0.69, at rotation.

Another interesting question is: What is the effect on propeller efficiency during the takeoff run if we consider a turbonormalized TNIO-550B in an A-36 at 3600 lbs at 10,000 ft? At 3600 lbs the takeoff speed is 72 KEAS. At 10,000 ft 72 KEAS corresponds to a true airspeed of 83.8 KTAS. Thus, compared to a takeoff speed of 70 KEAS (81.5 KTAS) for the E33A the propeller advance ratio at takeoff for the A-36 increases as the ratio of the true airspeeds, i.e., by $83.8/81.5 = 1.028$ or by 2.8%. The power coefficient increases by the ratio of the BHPs, i.e., by $300/285 = 1.053$, or by 5.3% The power coefficient is now $C_p = 0.0783$ for the TNIO-550B. The detailed results are not explicitly shown in Figure 1. However, just looking at Figure 1 we can easily see that the efficiency for the TNIO-550B A-36 is not much different from that for the TNIO-520BB E33A.

The next article will look at propeller efficiency during various cruise climb scenarios.

† Here, for convenience, we assume that EAS = CAS = IAS. This is reasonable because compressibility effects are small and for the Bonanza the pitot-static/position errors are also small.