Range

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As I sit here in a B767 with my knees and nose in close proximity at 33,000 feet over the North Atlantic, I can't help thinking how much more comfortable I'd be in a Bonanza. There is one problem though — range — specifically not enough of it, unless we want to make multiple hops and go the long way around through Goose Bay, Greenland, Iceland and Scotland. Range is defined as how far the aircraft can fly with the fuel aboard. How we maximize the range is an important question.

Generally, most of us consider the fuel on board as available flight time plus a reserve for legal or contingency purposes. For example, if we normally cruise at seven or eight thousand feet at a true airspeed of 167 knots, burning 14 gph, and maintain a one hour fuel reserve, then, assuming we use three gallons of fuel for start up, taxi and take off, with 74 gallons of usable fuel aboard initially we have slightly over four hours (4.07 hrs) of flight time available for a no wind range of 680 nm, not counting the fuel used for climb and descent. Throw in a 20 knot headwind (I always have a headwind in both directions), and the available 'range' is reduced to 598 nm. However, this procedure does not yield the maximum range of the aircraft.

There are several techniques for determining the maximum range of a normally aspirated (nonturbocharged) piston-propeller aircraft like the Bonanza. If you want the best range while maintaining a specific percentage of available power, then you should fly at the maximum altitude at which you can maintain that percentage of power. This is the critical altitude for that percentage of available power. For example, from the POH, an E33A can maintain 65% power up to 7,600 feet. At that altitude the true airspeed (TAS) is 164 kts. Allowing for start up, taxi and climb and a 45 minute reserve at 45% power, the range is approximately 778 nm with an initial fuel load of 74 gallons. At a lower altitude, say 5,000 feet, the TAS is 160.5 kts, with a range of 767 nm under similar conditions. Notice that the range is decreased at an altitude lower than the critical altitude.

Fundamentally, this decrease in range occurs because for constant thrust power as the altitude increases the drag decreases and the power required curve slides to the right and upward, as shown in Fig. (1). Another way of saying this is that the drag decreases faster than the power required increases. The result is an increase in TAS and hence an increase in range. Above the critical altitude, the available thrust power decreases while the power required continues to increase. This results in a decrease in TAS and hence in range. Thinking about the problem for a bit shows that, at full power, maximum range occurs at sea level, because a normally aspirated engine cannot maintain full power above sea level on a standard day.

However, this is still not the maximum range of the aircraft. The maximum range of the aircraft is given by the so called Breguet equation

$$R = 326\frac{L}{D}\frac{\eta}{c}\ln\frac{W_0}{W_1}$$

where

R is the range

$\frac{L}{D}$ is the lift to drag ratio

η is the propeller efficiency

c is the specific fuel consumption in lb/bhp-hr

W_0 is the weight at the beginning of cruise

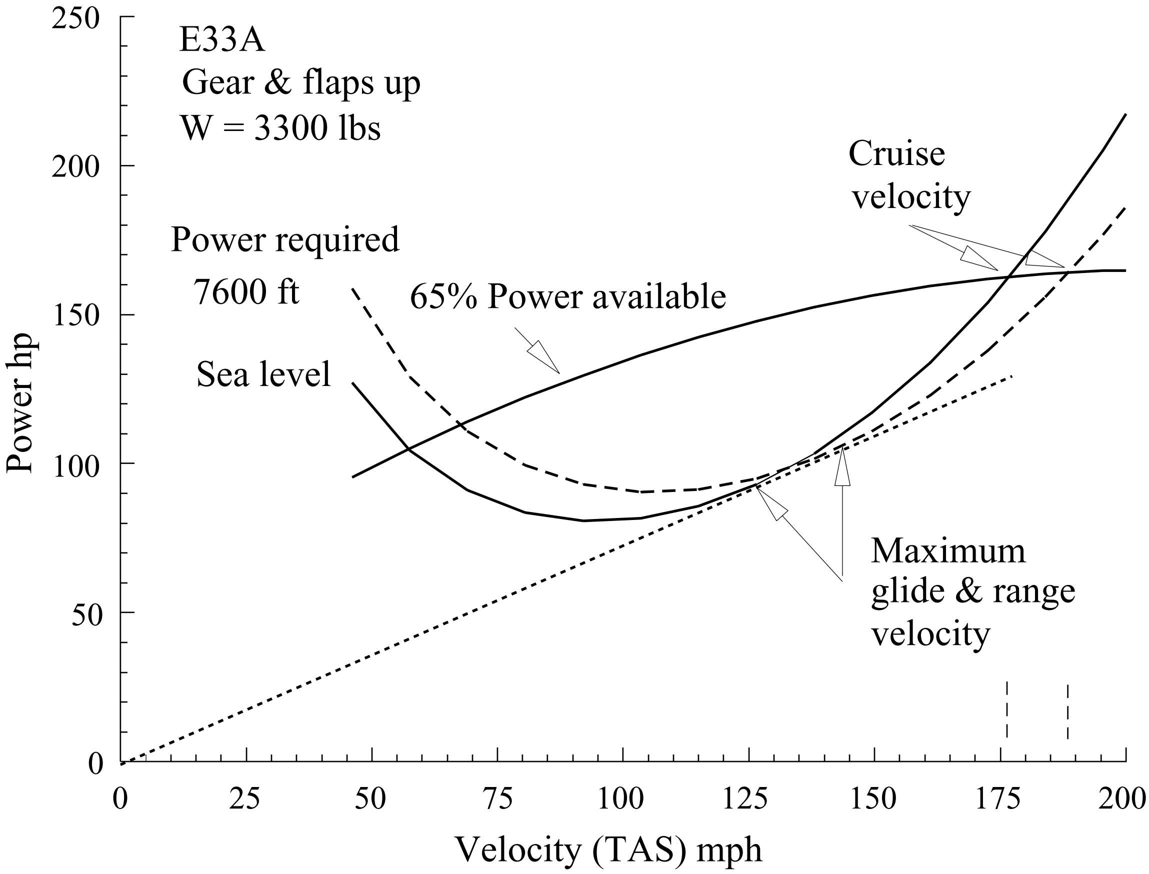

Figure 1. Power required and 65% power available as a function of true airspeed at sea level and 7600 feet.

W_1 is the weight at the end of cruise

and 326 is a conversion factor. ln means the natural logarithm, in this case of the ratio of W_0/W_1. You need a scientific calculator (or a table of logarithms) to determine the natural logarithm but here is a small table of appropriate values.

W_0/W_1	$\ln W_0/W_1$
1.03	0.02956
1.04	0.03922
1.05	0.04879
1.06	0.05827
1.07	0.06766
1.08	0.07696
1.09	0.08618
1.10	0.09531
1.11	0.10436

W_0/W_1	$\ln W_0/W_1$
1.12	0.11333
1.13	0.12222
1.14	0.13103
1.15	0.13976
1.16	0.14842
1.17	0.15700
1.18	0.16551
1.19	0.17395
1.20	0.18232

From the Breguet equation we see that to maximize the range we must minimize the specific fuel consumption, c, maximize the propeller efficiency, η, and maximize the lift to drag ratio, L/D. Minimizing the specific fuel consumption means that we must properly lean the engine. Maximizing the propeller efficiency means we must select an appropriate RPM for the flight velocity (more about this another time).

Maximizing the lift to drag ratio requires that we fly at the velocity for maximum L/D or $V_{L/D_{\max}}$, i.e., the best glide velocity which we discussed in a previous article (July/August 1995, p. 13). Hence, the velocity for maximum range is the best glide velocity. This velocity (TAS) is given by

$$\mathrm{TAS_{L/D}} = V_{\mathrm{L/D_{max}}} = \Big(\frac{2}{\sigma\rho_{\mathrm{SL}}}\,\frac{W}{b}\,\frac{1}{\sqrt{\pi f e}}\Big)^{1/2}$$

or, rewriting using the equivalent airspeed (EAS), we have

$$\mathrm{EAS_{L/D_{max}}} = \sqrt{\sigma}\,\mathrm{TAS_{L/D_{max}}} = \Big(\frac{2}{\rho_{\mathrm{SL}}}\,\frac{W}{b}\,\frac{1}{\sqrt{\pi f e}}\Big)^{1/2}$$

Notice that the EAS for maximum range does not depend on altitude. Hence, for a given weight and aircraft configuration it is constant. However, also notice that the maximum range EAS does vary with the weight. Specifically, it decreases with decreasing weight. The value given in the POH is for the maximum gross weight. For a model E33A at 3300 lbs the POH gives 122 mph as the speed for maximum range. How serious is the variation with weight? Starting at gross weight, from full tanks to dry tanks the weight of an E33A decreases by about 14%, which yields a decrease in the maximum range EAS by about 7%, or approximately 8.5 mph to 113.5 mph. However, this does not mean a decrease in maximum range by 7%. The maximum lift to drag ratio vs velocity curve is very flat in this region.

To see this, consider the power required vs velocity curve shown in Fig. (1). Recalling that power is thrust times velocity, $P = TV$, or that $T = P/V$, what we want to do is minimize the power required while simultaneously maximizing the velocity. This condition is obtained by drawing a line through the origin of the P_R vs V curve just tangent to (touching) the P_R curve, as shown by the dashed line in Fig. (1). Notice how 'flat' the P_R vs V curve is in this region. Hence, flying a few mph too fast or too slow compared to the maximum range velocity decreases the total range by only a small amount.

More important is the thrust power required to achieve the maximum range velocities. For maximum range the thrust power required is very low, ranging from 39% at sea level to 43% at 10,000 feet, and may result in manifold pressure and RPM combinations outside the manufacturer's recommended continuous operating range, assuming that the manufacturer even gives power settings this low. Notice again that flying at the critical altitude for the given power desired yields the maximum range. For these low powers, the critical altitude is typically above the oxygen levels. Thus, in practice, without oxygen, we are typically limited to 10,000 feet. So, if you need the extra range climb up there, throttle back and enjoy. Besides, in the summer it is cooler and you might even find a tailwind!