Weight Effects
Part 1
David F. Rogers

Most of us normally operate our aircraft at less than gross weight, yet weight significantly affects the performance of the aircraft in a number of ways. For example, rate-of-climb, the velocity for maximum rate-of-climb, the velocity for maximum climb angle, stall velocity, best glide velocity, the velocity for minimum sink rate are all affected.

Again, we recall the familiar equation for the power required to maintain level flight

$$P_r = \underbrace{\frac{\sigma\rho_{\text{SL}}}{2} f V^3}_{\text{parasite}} + \underbrace{\frac{2}{\sigma\rho_{\text{SL}}}\frac{1}{\pi e}\left(\frac{W}{b}\right)^2 \frac{1}{V}}_{\text{effective induced}}$$

First, let's take a look at the effect of weight on cruise velocity. We all know that the lighter the aircraft the faster we fly. But, by how much? Looking at the equation we see that the parasite power required term does not include the weight. Notice also that the velocity appears cubed, i.e., $V \times V \times V$. When you cube the cruise velocity you get a very large number! However, the weight appears in the effective induced power required term; but notice that the velocity now appears in the denominator (on the bottom). Thus, you divide by a large number which results is a very small number. Consequently, the contribution of the effective induced power required term is so much smaller than the contribution of the parasite power required term that we can neglect it. But this means that cruise velocity essentially does not depend on the weight!

Figure 1, which shows power required curves for a model E33A at weights from 2700 to 3300 pounds, along with the power available curves for 65%, 75% and 100% power for a three-bladed propeller, confirms this result. Recalling that the cruise velocity is given by the intersection of the power required and power available curves, note that Figure 1 also shows the variation with weight of the cruise velocity for 65% and 75% power. Since neglecting the contribution of the effective induced power required term is only an approximation, there is, of course, a small variation with weight, but it is very small. The decrease in maximum and cruise velocity is approximately one statute mile per hour per 100 pounds of weight increase for 65%, 75% and 100% power.

Now let's look at the variation of the velocity for maximum lift to drag ratio, $V_{L/D_{\text{max}}}$, with weight. The velocity for maximum lift to drag ratio is also the best glide velocity and, as we shall show in a subsequent article, the velocity for best range. Recall that

$$V_{\text{L/D}_{\text{max}}} = \left(\frac{2}{\sigma\rho_{\text{SL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\right)^{1/2} = \text{Constant}\sqrt{W}$$

Thus, the best glide velocity decreases as the weight decreases. Recall that the best glide velocity is given by the tangent (the line just touching) to the power required curve as shown by the dashed line in Figure 1. Typically the POH gives only the velocity for maximum gross weight. Figure 2 shows the ratio of the best glide velocity at reduced weight to that at maximum gross weight. The best glide velocity decreases approximately 1.5% for each 100 pounds below gross weight. For an E33A, the maximum gross weight best glide velocity is 122 mph. At 2500 lbs, which represents a single pilot with approximately one hour of fuel in the tanks, the best glide velocity is approximately 107 mph, a significant difference.

By the way what about this square root stuff? I can't do square roots in my head! Actually there is an easy approximation that you *can* do in your head with a little bit of effort. Calculating

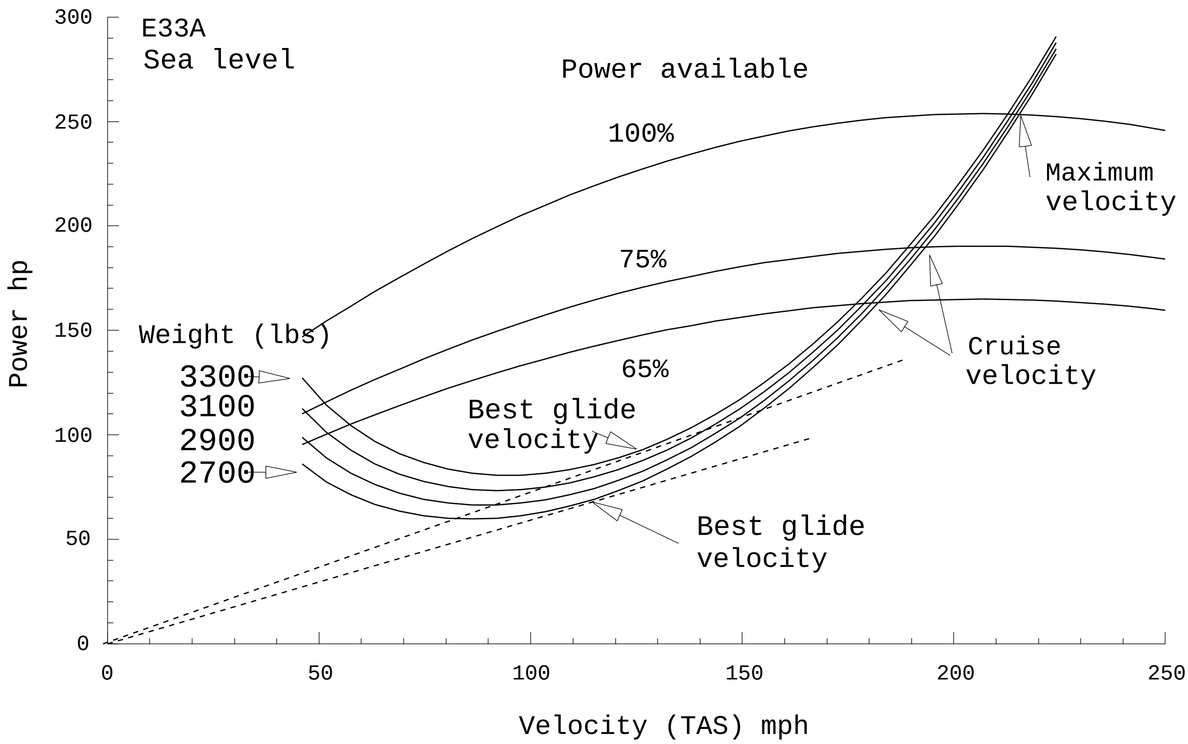

Figure 1. Power available and power required vs velocity.

the percentage reduction in weight (or whatever) and dividing by two gives the percentage reduction in the velocity (or whatever). Here, let's try it: $(3300-2500)/3300 = 0.24$, or about 24%, $(3300-2500 = 800$ and $800/3300$ is approximately $800/3200$ which is 0.25, but we are really dividing by 3300 which is a bit larger than 3200 so make it a bit less, or 0.24). Dividing by two gives 12%, or a 12% reduction in best glide velocity. Now 12% of 122 mph is about 14 mph (well 10% of 122 mph is about 12 mph and 2% is about 2 mph and $12+2$ is 14 mph) or a best glide velocity of about $122-14 = 108$ mph, which is close enough.

The velocity for minimum power required is also the velocity for maximum endurance and the velocity for minimum sink rate. The equation is

$$V_{P_{r_{\min}}} = \left(\frac{4}{3\pi f e}\right)^{1/4}\sqrt{\frac{1}{\sigma\rho_{\text{SL}}}}\sqrt{\frac{W}{b}} = \text{Konstant}\sqrt{W}$$

which is the same variation with weight as that for the best glide velocity. This is not surprising, since we recall that

$$\frac{V_{P_{r_{\min}}}}{V_{L/D_{\max}}} = 0.76$$

Consequently, Figure 2 also applies to the variation of the velocity for minimum sink rate and to the velocity for maximum endurance, as we shall show in a subsequent article.

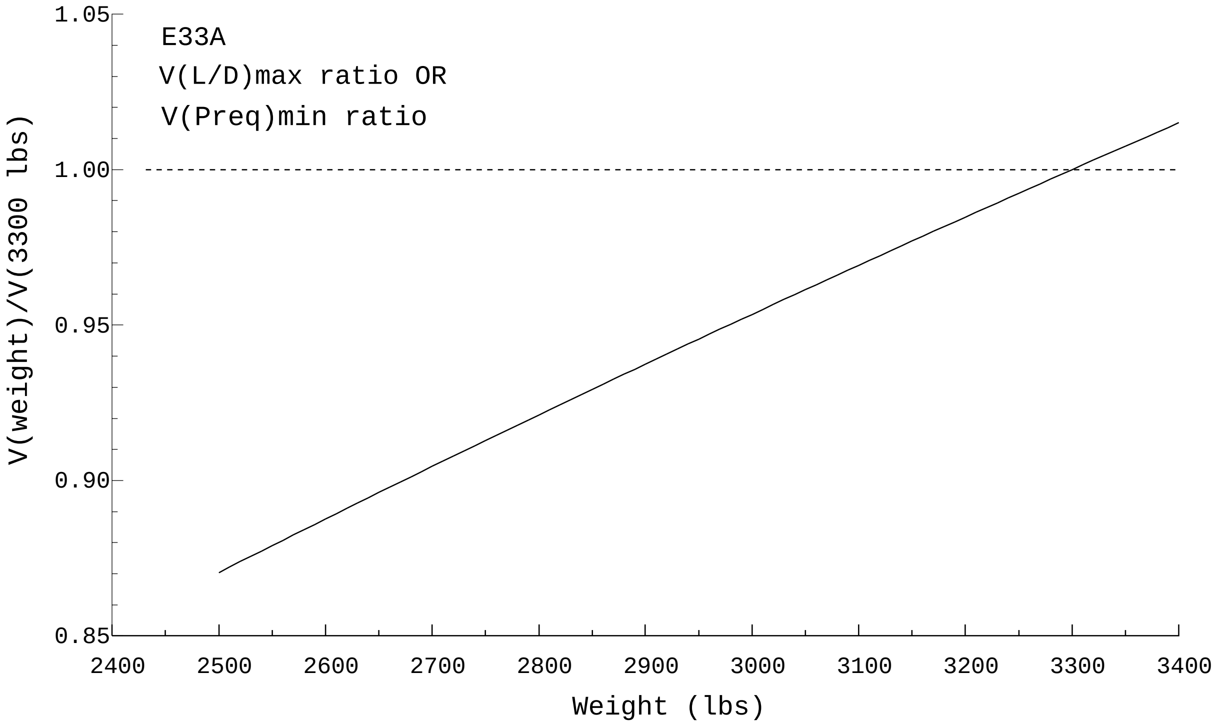

Figure 2. Velocity ratio vs weight.

The effect of weight on the rate-of-climb is not as straight forward. Recall that the equation for the rate-of-climb is

$$\text{Rate-of-Climb} = \frac{\text{Power Available}-\text{Power Required}}{\text{Weight}}$$

but recall from above that the power required also depends on weight, in fact upon the square root of weight. Thus, no simple equation shows us the effect. What is obvious is that the rate-of-climb *increases* as the weight *decreases*. So keep it light whenever rate-of-climb is critical. Doing the calculations and plotting the results for a model E33A, which has a maximum gross weight of 3300 lbs yields Figure 3. Figure 3 contains two graphs. The graph in the upper right corner shows the effect of weight on the absolute rate-of-climb for weights from 2400 to 3300 lbs and for altitudes from sea level to 16,000 feet. The main graph shows the *ratio* of the rate-of-climb at some weight to that at 3300 lbs. For example, at 16,000 feet the rate-of-climb at 3000 lbs compared to that at 3300 lbs is a factor of approximately 1.7 larger. However, notice from the upper right graph that it is still only about 400 fpm. This is not surprising since the POH gives the service ceiling, where the rate-of-climb is 100 fpm, for an E33A at full gross weight (3300 lbs) as 18,300 feet.

The velocity for best rate-of-climb *decreases* with altitude while the velocity for best climb angle, for example as used in an obstacle clearance takeoff, *increases* with altitude, as shown in Figure 4 for a model E33A at full gross weight. Notice that they meet at the absolute ceiling of the aircraft, where the rate-of-climb is zero, i.e., at about 20,000 feet. This means that at the

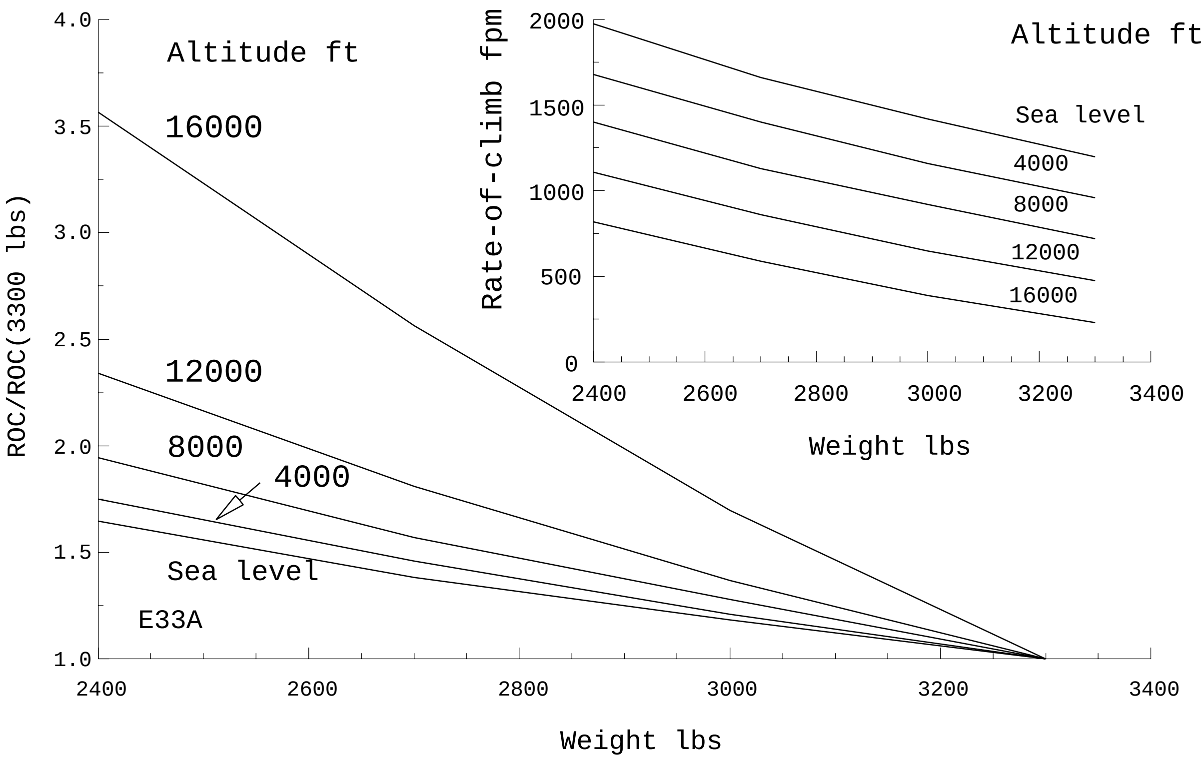

Figure 3. Rate-of-climb ratio vs weight.

absolute ceiling the aircraft can maintain steady level flight at only this *one* velocity of about 97 mph indicated airspeed.

How can we estimate the effect of weight on the velocity for maximum rate-of-climb? The actual calculation is a bit complex, so let's use an approximation. Looking back at Figure 1 and recalling that the rate-of-climb depends on the difference between the power available and the power required to maintain steady level flight, notice that the largest difference between these two curves occurs approximately at the velocity for minimum power required. Thus, the velocity ratio shown in Figure 2 also gives us an approximation for the effect of weight on the velocity for maximum rate-of-climb. For example, at 3000 lbs the velocity ratio from Figure 2 is approximately 0.95, and thus the velocity for maximum rate-of-climb will be approximately 5% less than that shown in Figure 4. Note that the approximation tends to overestimate the decrease in velocity for maximum rate-of-climb for weights less than about 2900 lbs. However, most of us seldom fly at those low weights relative to gross weight, so the approximation is good enough for a rule-of-thumb.

In another article we'll look at the effects of weight on range and endurance. As always, remember that the POH is the definitive authority on the performance of *your* specific aircraft.

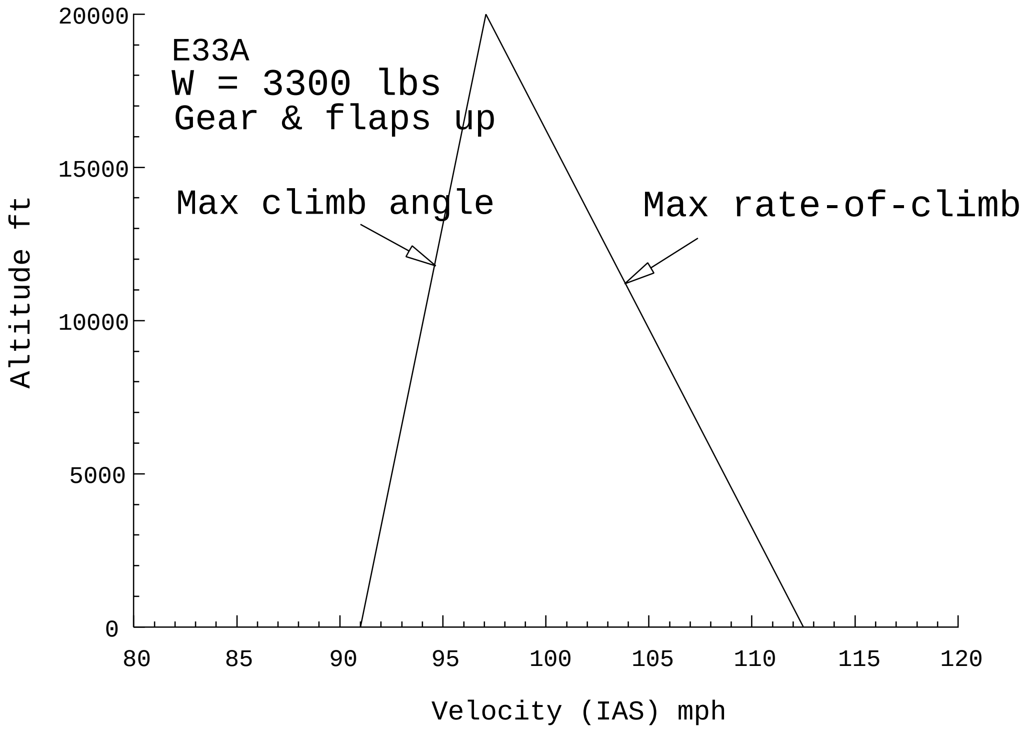

Figure 4. Altitude vs velocity for maximum rate-of-climb and maximum climb angle.

